Solution to MHT CET – 2021
20th September (Shift - 1)

Section I

PHYSICS

1. (C)

$T_1 = 2\pi\sqrt{\frac{m_1}{k}}$, $T_2 = 2\pi\sqrt{\frac{m_1 + m_2}{k}}$

$\therefore \frac{T_2}{T_1} = \sqrt{\frac{m_1 + m_2}{m_1}}$

$\therefore \frac{T_2^2}{T_1^2} = \frac{m_1 + m_2}{m_1}$

$\therefore \frac{T_2^2 - T_1^2}{T_1^2} = \frac{m_2}{m_1}$

2. (D)

Gyromagnetic ratio $= \frac{e}{2m_e} = \frac{A}{2}$

3. (D)

If dielectric of dielectric constant k is placed between the plates of a capacitor its capacitance increase k times. If it is removed the capacitance will become $\frac{1}{k}$ times.

4. (B)

$Y_1 = \frac{n\lambda_1 D}{d}$, $Y_2 = \frac{n\lambda_2 D}{d}$

$\therefore \frac{Y_1}{Y_2} = \frac{2\lambda_1}{\lambda_2}$

5. (C)

$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1}\right)^4$

$\therefore 16 = \left(\frac{T_2}{T_1}\right)^4 \quad \therefore \frac{T_2}{T_1} = 2$

6. (B)

Since the clockwise current is increasing, the induced current in the loop will be such as to oppose the change in magnetic flux and hence it will flow in anticlockwise direction.

7. (B)

$x = x_1 + x_2 = A\sin(\omega t + \alpha)$

Maximum value of $x_1 + x_2 = A = \sqrt{2}a$

$A^2 = a^2 + a^2 + 2a^2\cos\alpha = 2a^2(1 + \cos\alpha)$

$\therefore 2a^2 = 2a^2(1 + \cos\alpha) \quad \therefore 1 = 1 + \cos\alpha$

$\therefore \cos\alpha = 0 \quad \therefore \alpha = \frac{\pi}{2}$

8. (D)

A dipole has two equal and opposite charges. In a uniform electric field, the two charges will experience equal and opposite force, hence net force will be zero.

The potential energy of an electric dipole is given by

$U = -pE\cos\theta$

where θ is the angle between the dipole moment and the electric field. The potential energy will be minimum when cosθ is maximum.

For $\theta = 0°$, cosθ has maximum value equal to 1.

$\therefore\ U = -pE$ when $\theta = 0°$.

9. (C)

$n_A - n_B = n_1$ ...(i)

$n_A - n_C = n_2$ ...(ii)

Subtracting Eq.(ii) from Eq.(i)

$n_C - n_B = n_1 - n_2$

or $n_B - n_C = n_2 - n_1$

10. (C)

The magnetic field

$B = \mu_0 H$

For a solenoid $B = \mu_0 nI$ $\therefore$ $H = nI$

11. (B)

For an adiabatic process at constant pressure we have

$TV^{\gamma-1} = \text{constant}$ ...(1)

Given $T \propto \frac{1}{\sqrt{V}}$

$\therefore$ $TV^{1/2} = \text{constant}$

By (1) and (2) $\gamma - 1 = \frac{1}{2}$

$\therefore\ \gamma = 1.5$

12. (C)

In Celsius scale the freezing and boiling points are 0°C and 100°C. In the given imaginary scale the freezing and boiling points are 39°W and 239° W. Hence we can write

$$\frac{C}{100} = \frac{W-39}{200}$$

For $C = 39°$, $\frac{39}{100} = \frac{W-39}{200}$

Solving, $W = 117°C$

13. (D)

When a d.c. voltage is applied only the resistance of the coil comes into play, its inductive reactance is zero.

$\therefore\ R = \frac{V}{I} = \frac{200}{1} = 200\,\Omega$

When a.c. voltage is applied, the resistance and inductive reactance come into play and the coil has an impedance Z.

The impedance $Z = \frac{V}{I} = \frac{200}{0.5} = 400\,\Omega$

$$Z^2 = R^2 + X_L^2$$

$$\therefore \quad (400)^2 = (200)^2 + X_L^2$$

$$\therefore \quad X_L^2 = (400)^2 - (200)^2 = 12 \times 10^4$$

$$\therefore \quad X_L = 2\sqrt{3} \times 10^2 = 200\sqrt{3}\,\Omega \qquad \ldots(1)$$

Also $X_L = 2\pi f L = 2\pi f \times \frac{2\sqrt{3}}{\pi} = 4\sqrt{3}f \qquad \ldots(2)$

By (1) and (2) : $4\sqrt{3}f = 200\sqrt{3}$

$\therefore \qquad f = 50$ Hz

14. (C)

Power $P = \frac{\text{Energy}}{t} = \frac{nh\nu}{t} = \frac{nhc}{\lambda t}$

$$\therefore \quad n = \frac{P\lambda t}{hC}$$

15. (B)

Excess pressure in a soap bubble is given by

$$P = \frac{4T}{R}.$$

Hence if radius is increased, the pressure will decrease.

16. (D)

$$y = 10\sin\left(\frac{2\pi t}{30} + \alpha\right)$$

At $t = 0$, $y = 5$ cm $\quad \therefore \; 5 = 10 \sin\alpha$

$\therefore \; \sin\alpha = \frac{1}{2} \quad \therefore \; \alpha = \frac{\pi^c}{6}$

At $t = 7.5$, the total phase $\phi = \frac{2\pi \times 7.5}{30} + \alpha$

$$= \frac{\pi}{2} + \alpha$$

$$= \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3} \text{ rad}$$

17. (D)

If q is the charge on the ring, then the potential at the centre is given by

$$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{R}$$

But $q = \sigma \times \pi R$

$$\therefore \; V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\rho\pi R}{R} = \frac{\sigma}{4\varepsilon_0}$$

18. (C)

If p is the number of loops (or antinodes) then we have,

$Tp^2 = \text{constant}$ where T is the tension

$\therefore\ T_1P_1^2 = T_2^2P_2^2$

$\therefore\ \frac{T_2}{T_1} = \frac{P_1^2}{P_2^2} = \left(\frac{4}{2}\right)^2 = 4 \quad \therefore\ T_2 = 4T_1 = 4 \times 1 = 4\ \text{kg–wt}$

19. (B)

The volume of the bigger drop will be equal to the sum of the volumes of the smaller drops

$\therefore\ \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_1^3 + \frac{4}{3}\pi R_2^3$

$\therefore\ R = \left(R_1^3 + R_2^3\right)^{1/3}$

20. (B)

The resistance of the galvanometer and the shunt are in parallel. Their equivalent resistance is given by

$$\frac{1}{R} = \frac{1}{G} + \frac{1}{S}$$

$$\therefore\ \frac{1}{S} = \frac{1}{R} - \frac{1}{G}$$

$$\therefore\ \frac{1}{S} = \frac{1}{2.5} - \frac{1}{50} = \frac{19}{50} \quad \therefore\ S = \frac{50}{19}\Omega$$

21. (D)

Initially the electron is in the third excited state i.e. n = 4. Hence we can have transitions :

$4 \to 3,\ 4 \to 2,\ 4 \to 1$

$3 \to 2,\ 3 \to 1$

$2 \to 1$

Hence six transitions are possible.

22. (D)

In AND gate, output is 1 only if both the inputs are '1'. Hence output will be '0' for (0, 1) and (1, 0)

In NOR gate, output is '1' only if both the inputs are '0'. Hence output will be '0' for (0, 1) and (1, 0)

23. (B)

The induced emf is given by

$e = B\ell v = 1.2 \times 0.6 \times 10 = 7.2\ \text{V}$

24. (A)

The potential energy of electron in hydrogen atom is given by

$$U = -\frac{1}{4\pi\varepsilon_0} \cdot \frac{e^2}{r}$$

Hence as r increases, potential energy will increase. (It will become less negative)

Alternatively, it is also given by

$$U = -\frac{13.6}{n^2}\ \text{eV}$$

25. (A)

$m = 5\text{ g} = 5 \times 10^{-3}\text{ kg}, A = 0.3\text{ m}, T = \frac{\pi}{5}\text{ s}$

$\omega = \frac{2\pi}{T} = 10\text{ rad/s}$

Maximum force $F = m\omega^2 A = 5 \times 10^{-3} \times (10)^2 \times 0.3 = 0.15\text{ N}$

26. (D)

$W_1 = 620\text{ N}, W_2 = 340\text{ N}$

$W_1 = m(g + a)$ ...(i)

$W_2 = m(g - a)$ ...(ii)

$\therefore \frac{W_1}{W_2} = \frac{620}{340} = \frac{g+a}{g-a}$

$\therefore \frac{31}{17} = \frac{g+a}{g-a}$

Solving : $a = \frac{7}{24}g$

Putting this value in Eq. (i) and solving we get $mg = 480\text{ N}$

27. (A)

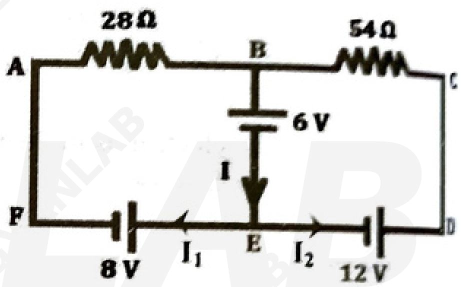
Applying Kirchoff's voltage law to loop ABEFA we get

$-28 I_1 - 6 - 8 = 0$

$-28 I_1 - 14 = 0$

$\therefore -28 I_1 = 14$

$\therefore I_1 = \frac{-14}{28} = -\frac{1}{2}\text{ A}$



Applying KVL to BCDEB

$54 I_2 - 12 + 6 = 0$

$\therefore 54 I_2 = 6$

$\therefore I_2 = \frac{6}{54} = \frac{1}{9}\text{ A}$ $\quad \therefore I = I_1 + I_2 = -\frac{1}{2} + \frac{1}{9} = -\frac{7}{18}\text{ A}$

28. (A)

For uniform velocity, the distance travelled is given by

$x = vt$

For uniform acceleration $x = \frac{1}{2}at$

$\therefore \frac{1}{2}at^2 = Vt$

$\therefore t = \frac{2V}{a}$

29. (C)

Width of central maximum $= \frac{2\lambda D}{a}$

$\therefore a = \frac{2\lambda D}{a} \quad \therefore D = \frac{a^2}{2\lambda}$

30. (C)

Since the coils are connected to a dc voltage. Inductive reactance will be zero, only resistance has to be considered.

When the coil is broken into two identical points, each part will have resistance $\frac{R}{2}$. When these are connected in parallel, their equivalent resistance will be $\frac{R}{4}$. Hence the current I is given by

$$I = \frac{E}{R/4} = \frac{4E}{R}$$

31. (B)

Frequency of vibration n will be same for both the segments. If p_1 and p_2 are the number of loops for the wires of radius r and 2r then we have

$$n = \frac{p_1}{2\ell r}\sqrt{\frac{T}{\pi\rho}} = \frac{p_2}{4\ell r}\sqrt{\frac{T}{\pi\rho}} \qquad \therefore \frac{p_1}{p_2} = \frac{1}{2}$$

32. (B)

Since reflected and refracted rays are perpendicular to each other,

$$r + r_1 = 90°$$

$$\therefore r_1 = 90° - r$$

Refractive index of the denser medium

$$\mu = \frac{\sin r_1}{\sin r} = \frac{\sin(90° - r)}{\sin r} = \frac{\cos r}{\sin r} = \cot r$$

Also, $\mu = \frac{1}{\sin C} \qquad \therefore \frac{1}{\sin C} = \cot r$

or $\sin C = \frac{1}{\cot r} = \tan r = \tan i$

$\therefore \quad i = \tan^{-1}(\sin C)$

33. (D)

$$\gamma = \frac{C_p}{C_v}, \text{ Also } C_p - C_v = R$$

$$C_p = \gamma C_v$$

$$\therefore \gamma C_v - C_v = R \quad \therefore C_v(\gamma - 1) = R \qquad \therefore C_v = \frac{R}{\gamma - 1}$$

34. (B)

Force due to surface tension $F = 2\pi r T$

$$= 2 \times \frac{22}{7} \times 2 \times 10^{-3} \times 70 \times 10^{-3}$$

$$= 8.8 \times 10^{-3} \text{ N}$$

35. (C)

Frequency = N

For each wave, a positive pulse is given as output

Hence N output pulses are given out.

36. (A)

The image is real and hence inverted.

$\therefore \quad \frac{v}{u} = -n \text{ or } u = -\frac{v}{n}$

By lens equation, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$\therefore \quad \frac{1}{v} + \frac{n}{v} = \frac{1}{f}$

$\therefore \quad \frac{1+n}{v} = \frac{1}{f}$

$\therefore \quad v = f(1+n)$

37. (C)

Moment of inertia of the solid sphere $I = \frac{2}{5}MR^2$

Moment of inertia of the disc about an axis passing through its edge and perpendicular to the plane is given by

$$I' = \frac{MR'^2}{2} + MR'^2 = \frac{3}{2}MR'^2$$

$I' = I \quad \therefore \frac{3}{2}MR'^2 = \frac{2}{5}MR^2$

$\therefore \quad R'^2 = \frac{4}{15}R^2 \qquad \therefore R' = \frac{2}{\sqrt{15}}R$

38. (A)

$$g' = g\left(1 - \frac{d}{R}\right)$$

If $g' = \frac{g}{n}$ then $\frac{g}{n} = g\left(1 - \frac{d}{R}\right)$

$\therefore \quad \frac{1}{n} = 1 - \frac{d}{R}$

$\therefore \quad \frac{d}{R} = 1 - \frac{1}{n} = \frac{n-1}{n} \qquad \therefore \quad d = R\left(\frac{n-1}{n}\right)$

39. (C)

Resistivity of conductors increases with temperature and that of semiconductor decreases with temperature.

When temperature is decreased, the resistivity of copper will decrease while that of silicon will increase.

40. (B)

$E_1 = \frac{1}{2}I_1\omega_1^2$

$E_2 = \frac{1}{2}I_2\omega_2^2 = \frac{1}{2}\cdot 3I_1 \cdot \omega_1^2 \qquad [\because \; I_2 = 3I_1]$

$= \frac{3}{2}I_1\omega_2^2$

$E_1 = 27E_2 = 27 \times \frac{3}{2} \times I_1\omega_2^2$

$\therefore \ \frac{1}{2}I_1\omega_2^2 = \frac{81}{2}I_1\omega_2^2$

$\therefore \ \omega_1 = 9\omega_2$

$\frac{L_1}{L_2} = \frac{I_1\omega_1}{I_2\omega_2} = \frac{I_1 \times 9\omega_2}{3I_1 \times \omega_2} = 3$

41. (A)

$E_1 = \frac{hc}{\lambda_1} - W \quad \therefore \ E_1\lambda_1 = hc - W\lambda_1$

$E_2 = \frac{hc}{\lambda_2} - W \quad \therefore \ hc = E_1\lambda_1 + W\lambda_1 \quad \ldots(i)$

$\therefore \ E_2\lambda_2 = hc - W\lambda_2 \quad \therefore \ hc = E_2\lambda_2 + W\lambda_2 \quad \ldots(ii)$

By Eq.(i) and (ii)

$E_1\lambda_1 + W\lambda_1 = E_2\lambda_2 + W\lambda_2$

$\therefore \ E_1\lambda_1 - E_2\lambda_2 = W(\lambda_2 - \lambda_1)$

$\therefore \ W = \frac{E_1\lambda_1 - E_2\lambda_2}{\lambda_2 - \lambda_1}$

42. (A)

Magnetic field at the centre of a circular coil is given by

$B = \frac{\mu_0 I}{2r}$

In this case the current $I = \frac{e}{T}$

where T is the period of revolution

$T = \frac{2\pi r}{v}$

$\therefore \ I = \frac{eV}{2\pi r} \quad \therefore \ B = \frac{\mu_0 eV}{4\pi r^2}$

43. (C)

The orbital velocity near the surface of the earth

$V = \sqrt{\frac{GM}{R}}$

At an altitude $\frac{R}{2}$, the orbital velocity

$V' = \sqrt{\frac{GM}{R + \frac{R}{2}}} = \sqrt{\frac{2GM}{3R}} \quad \therefore \ \frac{V'}{V} = \sqrt{\frac{2}{3}} \quad \text{or} \quad V' = \sqrt{\frac{2}{3}}V$

44. (D)

Inductive reactance when 100 V, 50 Hz source is connected is given by

$X_L = \frac{V}{I} = \frac{100}{8} = 12.5\,\Omega$

Resistance $R = \frac{V}{I} = \frac{100}{10} = 10\,\Omega$

When 100 V, 40 Hz supply is connected the inductive reactance will get reduced
Since it is proportional to the frequency

$\therefore \quad \frac{X'_L}{X_L} = \frac{40}{50} = \frac{4}{5}$

$\therefore \quad X'_L = \frac{4}{5} X_L = \frac{4}{5} \times 12.5 = 10\,\Omega$

The resistance will remain same.

When they are connected in series the impedance will be given by

$$Z = \sqrt{R^2 + X'^2_L} = \sqrt{(10)^2 + (10)^2} = 10\sqrt{2}\ \text{A}$$

$$I = \frac{V}{Z} = \frac{100}{10\sqrt{2}} = 5\sqrt{2}\ \text{A}$$

45. (C)

$R = 0.4$ m, $M = 1$ kg, $\alpha = 10$ rad/s^2

Moment of inertia $I = \frac{MR^2}{2} = \frac{1 \times (0.4)^2}{2} = \frac{0.16}{2} = 0.08\ \text{kg m}^2$

Torque $\quad \tau = RF = I\alpha$

$\therefore \quad F = \frac{I\alpha}{R} = \frac{0.08 \times 10}{0.4} = 2\text{N}$

46. (D)

Heat supplied at constant pressure $Q_1 = nC_p dT$

Heat supplied at constant volume $Q_2 = nC_v\, dT$

Work done $\quad W = Q_1 - Q_2 = n(C_p - C_v)\, dT$

$\therefore \quad \frac{W}{Q_2} = \frac{C_p - C_v}{C_v} = \frac{C_p}{C_v} - 1 = \frac{5}{3} - 1 = \frac{2}{3}$

$\therefore \quad Q_2 = \frac{3W}{2}$

47. (A)

Distance between the first minima $= \frac{2\lambda D}{a} = \frac{2 \times 5 \times 10^{-7} \times 2}{0.2 \times 10^{-3}} = 10^{-3}$ m

48. (A)

Due to charge $-q$ at the centre of the shell, a charge q will be induced on the inner surface and $-q$ on the outer surface. The charge on outer surface will become $Q - q$. Hence surface charge densities will be $\frac{q}{4\pi r_1^2}$ and $\frac{Q-q}{4\pi r_2^2}$

49. (C)

Period $= \frac{1}{n}$

Time required for x vibrations, $t = \frac{x}{n}$

Distance travelled by the wave, $Vt = \frac{xV}{n}$

50. (B)

Force, F = charge × Electric field $= 2eE$

Acceleration $= \frac{\text{Force}}{\text{Mass}} = \frac{2eE}{4m} = \frac{eE}{2m}$

CHEMISTRY

51. (D)

$n_{urea} = \frac{12}{60} = 0.2$ and $n_{glucose} = \frac{36}{180} = 0.2$

Now, $n_{urea} = n_{glucose}$

$\therefore\ \pi_{urea} = \pi_{glucose}$

Hence, if these solutions are separated by a semipermeable membrane, there is no flow of solvent in either direction.

52. (C)

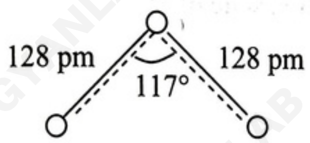
$$3I^-_{(aq)} + S_2O^{2-}_{8(aq)} \longrightarrow I^-_{3(aq)} + 2SO^{2-}_{4(aq)}$$

$$\therefore\ \frac{d[I_3^-]}{dt} = \frac{1}{2}\frac{d[SO_4^{2-}]}{dt} = \frac{1}{2}\times 0.022 = 0.011\ \text{mol dm}^{-3}\ \text{sec}^{-1}$$

53. (B)

Schiff test confirms the presence of aldehydic (–CHO) group.

54. (C)



Resonance hybrid of Ozone

O – O bond length in resonance hybrid of ozone = 128 pm

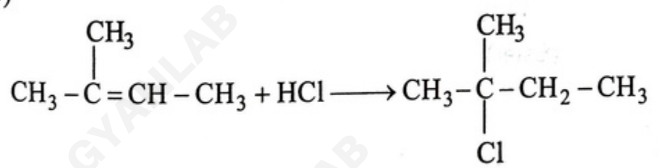
55. (D)

$$\text{Number of atoms} = \frac{\text{Mass}}{\text{Atomic mass}}\times N_A = \frac{3.9}{39}\times N_A = 0.1\ N_A$$

In BCC unit cell, n = 2

$$\therefore\ \text{Number of unit cells} = \frac{0.1\ N_A}{2} = \frac{N_A}{20}$$

56. (B)



2-Methylbut-2-ene   2–Chloro–2–methylbutane

57. (B)

Cd (Z = 48) ⇒ [Kr] $4d^{10}\ 5s^2$

Cd has completely filled 4d – orbital.

58. (A)

Aldehydes (except formaldehyde) on reaction with Grignard reagent followed by hydrolysis forms secondary alcohol.

$\therefore$ CH_3CHO forms secondary alcohol.

59. (D)

Formula of Crystalline chloride of lithium $\Rightarrow$ $LiCl.\ 2H_2O$

60. (A)

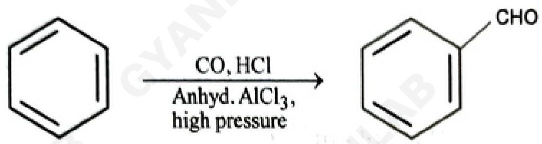
$AgCl_{(s)} \rightleftharpoons Ag^{+}_{(aq)} + Cl^{-}_{(aq)}$

Here, x = 1, y = 1

$\therefore\ Ksp = S^2 = (1.25 \times 10^{-5})^2 = 1.56 \times 10^{-10}$

61. (C)

Gatterman – Koch formylation of arene



Benzene   Benzaldhyde

62. (A)

$Pt[(NH_3)_2\ Cl_2]$ → It is a neutral complex. In this complex, chlorine atoms are in coordination sphere. Coordination number of Pt = 4, oxidation state of Pt = +2.

63. (D)

64. (A)

$\wedge = \dfrac{1000\,k}{c} \qquad \therefore\ k = \dfrac{\wedge c}{1000}$

Now, $\wedge = 106\ \Omega^{-1}\ cm^2\ mol^{-1}$, $c = 0.1\ mol\ L^{-1}$

$$\therefore\ k = \frac{106\,\Omega^{-1}\,cm^2\,mol^{-1} \times 0.1\,mol\,L^{-1}}{1000\,cm^3\,L^{-1}}$$

$$\therefore\ k = 1.06 \times 10^{-2}\,\Omega^{-1}\,cm^{-1}$$

65. **(D)**

$$Ar-OH + Cl-\underset{\underset{O}{\|}}{C}-R \xrightarrow{\text{Pyridine}} R-\underset{\underset{O}{\|}}{C}-O-Ar + HCl$$

Acid chloride   (Ester)

66. (C)

$P_1 = 105$ kPa, $V_1 = 11.2\ dm^3$

$P_2 = 210$ kPa, $V_2 = ?$

According to Boyle's law, $P_1V_1 = P_2V_2$

$$\therefore\ V_2 = \frac{P_1V_1}{P_2} = \frac{105\,kPa \times 11.2\,dm^3}{210\,kPa} = 5.6\ dm^3$$

67. (C)

68. (C)

$$H_2O_{(\ell)} + NH_{3(aq)} \rightleftharpoons NH^{+}_{4(aq)} + OH^{-}_{(aq)}$$

69. (A)

$$H_{2(g)} + \frac{1}{2}O_{2(g)} \longrightarrow H_2O_{(l)}$$

11.2 dm^3 of O_2 gives 18 g water at STP

$\therefore$ 9 g water is obtained from $\frac{11.2 \times 9}{18} = 5.6\ dm^3$ of O_2

70. (A)

$W_2 = 6$ g, $W_1 = 100$ g, $K_b = 0.52\ K\ kg\ mol^{-1}$, $T_b = 100.52°C$

$$\therefore \Delta T_b = T_b - T_b^{\circ} = (100.52 + 273) - (100 + 273)$$
$$= 0.52\ K$$

Now,

$$M_2 = \frac{1000 \cdot K_b \cdot W_2}{\Delta T_b \cdot W_1} = \frac{1000\ g\ kg^{-1} \times 0.52\ K\ kg\ mol^{-1} \times 6\ g}{0.52\ K \times 100\ g}$$

$\therefore$ $M_2 = 60\ g\ mol^{-1}$

71. (A)

Tert-Butyl alcohol has a much higher melting point because of its symmetrical structure.

72. (B)

73. (A)

$$NH_{3(g)} + HCl_{(g)} \longrightarrow NH_4Cl_{(s)}$$

In this reaction, gaseous reactants converted into solidified product which resulting in a decrease of volume and shows work of compression.

74. (B)

Undecane $\rightarrow C_{11}H_{24}$

75. (B)

$$\%\ \text{atom economy} = \frac{\text{Formula weight of the desired product}}{\text{Sum of formula weight of all the reactant used in the reaction}} \times 100$$

$$= \frac{46}{92} \times 100 = 50\ \%$$

76. (D)

For first order reaction,

$$k = \frac{0.693}{t_{\frac{1}{2}}} = \frac{0.693}{6.93} = 0.1\ \text{hour}^{-1}$$

Here, $[A]_0 = 100$, $[A]_t = 100 - 80 = 20$

Now, $k = \frac{2.303}{t} \log_{10} \frac{[A]_0}{[A]_t}$

$$\therefore t = \frac{2.303}{0.1} \log_{10} \frac{100}{20} = 23.03 \times \log_{10} 5 = 23.03 \times 0.699 = 16.10\ \text{hours.}$$

77. (B)

$$\underset{\text{Acetamide}}{CH_3-\overset{\overset{O}{\|}}{C}-NH_2} + 4[H] \xrightarrow[\text{or } Na/C_2H_5OH]{LiAlH_4/ether} \underset{\text{Ethylamine}}{CH_3-CH_2-NH_2}$$

78. (A)

$P_1 = 216$ mm Hg, $P_1^0 = 240$ mm Hg

$\Delta P = P_1^0 - P_1 = 240 - 216 = 24$ mm Hg

Now, $\dfrac{\Delta P}{P_1^0} = x_2$

$\therefore\ x_2 = \dfrac{24 \text{ mm Hg}}{240 \text{ mm Hg}} = 0.1$

Hence, mole fraction of solvent, $x_1 = 1 - x_2 = 1 - 0.1 = 0.9$

79. (A)

For f-orbital $\Rightarrow \ell = 3$

When, $n = 4,\ \ell = 3,\ m = 0$

$\Rightarrow$ The orbital is 4f.

80. (B)

pH = 2.34, c = 0.1 M

(i) $pH = -\log_{10}[H^+]$

$\log_{10}[H^+] = -pH = -2.34$

$= -2 - 0.34 - 1 + 1$

$= -3 + 0.66 = \bar{3}.66$

$\therefore\ [H^+] = \text{antilog } \bar{3}.66$

$= 4.571 \times 10^{-3}$ mol dm^{-3}

(ii) $[H^+] = \propto c$

$\therefore\ \propto = \dfrac{[H^+]}{c} = \dfrac{4.571 \times 10^{-3}}{0.1} = 4.571 \times 10^{-2}$

81. (C)

$$\underset{\text{Acetylene}}{C_2H_{2(g)}} + \frac{5}{2}O_{2(g)} \longrightarrow 2CO_{2(g)} + H_2O_{(\ell)} \quad ; \quad \Delta_rH^\circ = -1300 \text{ kJ mol}^{-1}$$

When 26 g of acetylene is completely burnt, the change in enthalpy = –1300 kJ

$\therefore$ For 39 g of acetylene, the change in enthalpy

$= \dfrac{-1300 \times 39}{26} = -1950$ kJ

82. (D)

When molten ionic compound is electrolyzed, a metal is deposited at cathode.

83. (C)

$R = 600\ \Omega$, $k_{KCl} = 0.0015\ \Omega^{-}\ cm^{-1}$, $c = 0.01$ M

Cell constant $= k_{KCl} \times R_{solution} = 0.0015\ \Omega^{-1}\ cm^{-1} \times 600\ \Omega = 0.90\ cm^{-1}$

84. (C)
$M = 180 \text{ g mol}^{-1}, \rho = 18 \text{ g cm}^{-3},$

For BCC crystal, n = 2, a = ?

$$\rho = \frac{M \times n}{a^3 \times N_A} \quad \therefore \quad a^3 = \frac{M \times n}{\rho \times N_A}$$

$$\therefore \; a^3 = \frac{180 \text{ g mol}^{-1} \times 2 \text{ atom}}{18 \text{ g cm}^{-3} \times 6.022 \times 10^{23} \text{ atom mol}^{-1}}$$

$$\therefore \; a^3 = 33.2 \times 10^{-24} \text{ cm}^3$$

$$\therefore \; a = \sqrt{33.2} \times 10^{-8} \text{ cm}$$

85. (A)
Selenium exists in two allotropic forms, red (non-metallic) and grey (metallic).

86. (B)
Multimolecular colloid – Gold sol
Macromolecular colloid – Nylon, Cellulose
Associated colloid – Soap.

87. (C)
Aromatic amines cannot be prepared by Gabriel phthalimide synthesis because aryl halides do not undergo nucleophilic substitution with the anion formed by phthalimide.

88. (D)

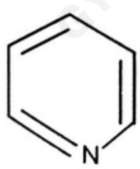
$$CH_3-\underset{\substack{|\\CH_3}}{CH}-CH_3 \xrightarrow[\text{UV light}]{Br_2} CH_3-\underset{\substack{|\\CH_3}}{CH}-CH_2-Br + HBr$$

2-Methylpropane → 1 - Bromo - 2 - Methylpropane

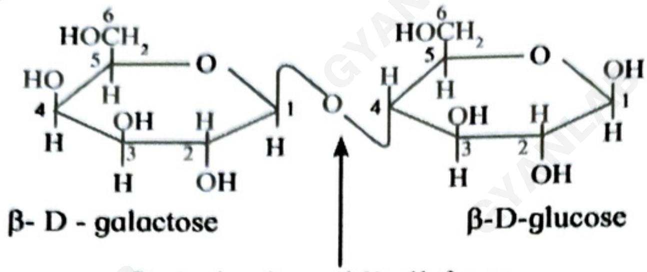
89. (C)
Ligands strength $\Rightarrow S^{2-} < H_2O < EDTA < en$

90. (C)

Pyridine is a heterocyclic compound.



91. (C)



β- 1, 4- glycosidic linkage

Haworth formula of lactose

92. (C)

93. (A)

Silver mirror test or Tollen's test is given by only aldehydes.

Hence, ethanal on reaction with ammonical silver nitrate solution forms precipitate of silver.

94. (C)

95. (C)

$V_1 = 10$ L, $V_2 = 5$L, W = 1730 J, T = 300 K, R = 8.314 J K^{-1} mol^{-1}, n = ?

$$W_{max} = -2.303\ nRT\ \log_{10}\frac{V_2}{V_1}$$

$$\therefore\ 1730\ J = -2.303\ n \times 8.314\ J\ K^{-1}\ mol^{-1} \times 300\ K \times \log_{10}\left(\frac{5}{10}\right)$$

$$\therefore\ n = \frac{1730\ J}{-2.303 \times 8.314\ J\ K^{-1}\ mol^{-1} \times 300K \times \log_{10}(0.5)}$$

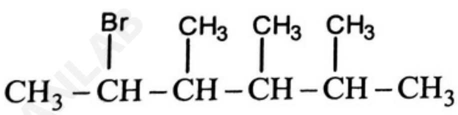
$$\therefore\ n = \frac{1730}{-5744.14 \times (-0.3010)}\ mol$$

$$\therefore\ n = \frac{1730}{1729} \approx 1\ mol$$

96. (D)

97. (C)

98. (B)



2 – Bromo – 3, 4, 5 – trimethylhexane

99. (A)

A more powerful nucleophile attacks the substrate faster and favours S_N2 mechanism.

100. (C)

$$\overset{+3}{C_2}O_4^{2-} + \overset{+7}{Mn}O_4^{-} + H^{+} \longrightarrow \overset{+2}{Mn^{2+}} + \overset{+4}{C}O_2 + H_2O$$

The oxidation number of C increases from +3 $\left(C_2O_4^{2-}\right)$ to +4 (CO_2) in above reaction.

$\therefore$ $\left(C_2O_4^{2-}\right)$ acts as a reductant.

Section II

MATHEMATICS

101.(A)

Let I $= \tan^{-1}(\sec x + \tan x)dx$

$$= \int \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)dx \quad = \int \tan^{-1} x\left[\frac{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)}\right]_{dx}$$

$$= \int \tan^{-1}\left(\frac{\cos\frac{x}{2}+\sin\frac{x}{2}}{\cos\frac{x}{2}-\sin\frac{x}{2}}\right)dx \quad = \int \tan^{-1}\left(\frac{1+\tan\frac{x}{2}}{1-\tan\frac{x}{2}}\right)dx$$

$$= \int \tan^{-1}\left[\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right]dx \quad = \int\left(\frac{\pi}{4}+\frac{x}{2}\right)dx$$

$$= \frac{\pi x}{4}+\frac{x^2}{4}+c$$

102.(C)

$$A(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} \Rightarrow |A(\alpha)| = \cos^2\alpha + \sin^2\alpha = 1$$

$$A^2(\alpha) = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\alpha-\sin^2\alpha & 2\sin\alpha\cos\alpha \\ -2\sin\alpha\cos\alpha & \cos^2\alpha-\sin^2\alpha \end{bmatrix} = \begin{bmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$\therefore \ adj[A^2(\alpha)] = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$$

$$\therefore \ \left[A^2(\alpha)\right]^{-1} = \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix} \quad \ldots[\because |A^2| = |A|^2 = 1]$$

$$\therefore \ [A^2(\alpha)]^{-1} = \begin{bmatrix} \cos(-2\alpha) & \sin(-2\alpha) \\ -\sin(-2\alpha) & \cos(-2\alpha) \end{bmatrix} = A(-2\alpha)$$

103.(B)

The sum of numbers on the two dice can be (2, 3, …, 12) and prime numbers in this list are 2, 3, 5, 7, 11.

2 $\Rightarrow$ (1, 1) and 3 $\Rightarrow$ (1, 2), (2, 1) and 5 $\Rightarrow$ (1, 4), (2, 3), (3, 2), (4, 1),

7 $\Rightarrow$ (1, 6), (2, 5), (3, 4), (4, 3), (5, 2), (6, 1) and 11 $\Rightarrow$ (5, 6), (6, 5)

Thus $n(s) = 6 \times 6 = 36$ and $n(E) = 15$

Hence required probability $= \frac{15}{36} = \frac{5}{12}$

104.(C)

As per data given, we write

$$\tan\frac{\pi}{4} = \left|\frac{2\sqrt{h^2 - ab}}{a+b}\right| = 1$$

Squaring both sides, we get

$$(a+b)^2 = 4(h^2 - ab)$$

$$\therefore\ 4h^2 = a^2 + 6ab + b^2$$

105.(B)

Let x be the one part and y be the other part.

We have $x + y = 20 \Rightarrow y = 20 - x$

As per condition given, we write

$$f(x) = (20 - x)x^3$$
$$= 20x^3 - x^4$$
$$\therefore\ f'(x) = 60x^2 - 4x^3$$

When $f'(x) = 0$, we get

$$4x^2(15 - x) = 0 \Rightarrow x = 0, 15$$
$$f''(x) = 120x - 12x^2$$
$$[f''(x)]_{x=15} = (120)(15) - (12)(15)^2 = -900 < 0$$

$\therefore$ f(x) is maximum when $x = 15$.

$\therefore\ y = 5 \Rightarrow xy = (15)(5) = 75$

106.(A)

Here equation of line OC is $y = x$ i.e. $x - y = 0$ and equation of line AB is $x = 5$ i.e. $x - 5 = 0$

Equation of line BC is $y = 3$ i.e. $y - 3 = 0$

Hence constraints for the shaded region are $x, y \ge 0, x - 5 \le 0, x - y \ge 0, y - 3 \le 0$

i.e. $x, y \ge 0, x \le 5, x - y \ge 0, y \le 3$

107.(C)

As per data given, we write

$$24 = \frac{1}{6}\left\{(\bar{a}+\bar{b})\cdot\left[(\bar{b}+\bar{c})\times(\bar{c}+\bar{a})\right]\right\}$$

$$= \frac{1}{6}\left\{(\bar{a}+\bar{b})\cdot\left[(\bar{b}\times\bar{c})+(\bar{b}\times\bar{a})+(\bar{c}\times\bar{a})\right]\right\} \quad \ldots[\because\ \bar{c}\times\bar{c} = 0]$$

$$\therefore\ 144 = \left[\bar{a}\cdot(\bar{b}\times\bar{c})\right]+\left[\bar{b}\cdot(\bar{b}\times\bar{c})\right]+\left[\bar{a}\cdot(\bar{b}\times\bar{a})\right]+\left[\bar{b}\cdot(\bar{b}\times\bar{a})\right]+\left[\bar{a}\cdot(\bar{c}\times\bar{a})\right]+\left[\bar{b}\cdot(\bar{c}\times\bar{a})\right]$$

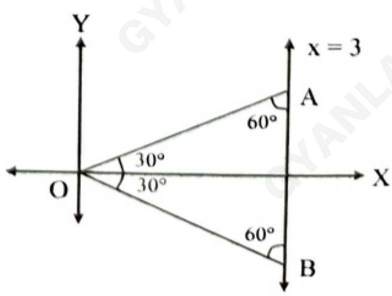
$$= \left[\bar{a}\cdot(\bar{b}\times\bar{c})\right]+0+0+0+0+\left[\bar{b}\cdot(\bar{c}\times\bar{a})\right]$$

$$= 2\left[\bar{a}\cdot(\bar{b}\times\bar{c})\right] \quad \ldots\left[\because\ \bar{b}(\bar{c}\times\bar{a}) = \bar{a}\cdot(\bar{b}\times\bar{c})\right]$$

$\therefore\ \bar{a}\cdot(\bar{b}\times\bar{c}) = 72$, i.e. volume of parallelepiped with coterminous edges $\bar{a}, \bar{b}, \bar{c}$.

108.(C)

Refer figure.



Slope of line OA = $\tan 30° = \frac{1}{\sqrt{3}}$ and

slope of line OB = $\tan(-30°) = \frac{-1}{\sqrt{3}}$

$\therefore$ Equation of OA is $y = \frac{1}{\sqrt{3}}x$ and equation of OB is $y = \frac{-1}{\sqrt{3}}x$

Hence required equation is

$\left(x-\sqrt{3}y\right)\left(x+\sqrt{3}\,y\right)=0$ i.e. $x^2-3y^2=0$

109.(A)

We have to find variance of numbers 2, 3, 4, …, 11.

Here $\bar{x} = \frac{2+3+4+...+11}{10} = \frac{65}{10} = 6.5$

Variance $= \frac{\Sigma\left(x_i-\bar{x}\right)^2}{n}$

$$= \frac{\left[(-4.5)^2+(-3.5)^2+(-2.5)^2+(-1.5)^2+(-0.5)^2+(0.5)^2+(1.5)^2+(2.5)^2+(3.5)^2+(4.5)^2\right]}{10}$$

$$= \frac{2(20.25+12.25+6.25+2.25+0.25)}{10} = \frac{41.25}{5} = 8.25$$

110.(D)

Here $k + 2k + 2k + 3k + k^2 + 2k^2 + 7k^2 + k = 1$

$\therefore\ 10k^2 + 9k - 1 = 0 \quad \Rightarrow (10k-1)(k+1) = 0$

$\therefore\ k = \frac{1}{10} \quad \ldots(k \geq 0)$

$\therefore\ F(4) = k + 2k + 2k + 3k = 8k = \frac{8}{10} = \frac{4}{5}$

111.(C)

The temperature T of the body will decrease with time.

The body is kept in a bath of temperature 32° F.

$\therefore\ \frac{dT}{dt}\alpha-(T-32) \quad \Rightarrow \frac{dT}{dt} = -k(T-32)$

112.(C)

Let $I = \int_0^{\pi/4} \log(1+\tan x)\,dx$

$$= \int_0^{\pi/4} \log\left[1+\tan\left(\frac{\pi}{4}-x\right)\right]dx$$

$$= \int_0^{\pi/4} \log\left[1+\left(\frac{1-\tan x}{1+\tan x}\right)\right]dx = \int_0^{\pi/4} \log\left(\frac{2}{1+\tan x}\right)dx$$

$$= \int_0^{\pi/4} (\log 2)\,dx - \int_0^{\pi/4} \log(1+\tan x)\,dx = \int_0^{\pi/4} (\log 2)\,dx - I$$

$\therefore\ 2I = (\log 2)[x]_0^{\pi/4} = (\log 2)\left(\frac{\pi}{4}\right) \quad \Rightarrow I = \left(\frac{\pi}{8}\right)\log 2$

113.(D)

$S = at^2 + bt + 6$

$\therefore\ V = \dfrac{dS}{dt} = 2at + b$ and $A = \dfrac{d^2S}{dt^2} = 2a$

When particle comes to rest,

$S = 16, t = 4, V = 0$

$\therefore\ 16 = a(4)^2 + b(4) + 6 \Rightarrow 16a + 4b = 10$ ...(1)

Also $0 = 2a(4) + b \Rightarrow b = -8a$ ...(2)

From (1) and (2), we get

$16a + 4(8a) = 10 \Rightarrow -16a = 10 \Rightarrow a = \dfrac{-5}{8}$

We have acceleration $A = 2a = 2\left(\dfrac{-5}{8}\right) = \dfrac{-5}{4}\ m/sec^2$

114.(B)

Let $p : x \in A \cap B$ and $q : x \in A$ and $x \in B$.

$\therefore$ Logical form of given statement is $p \rightarrow q$.

Now $p \rightarrow q \equiv \sim p \vee q$

$\therefore\ \sim(p \rightarrow q) \equiv \sim(\sim p \vee q) \equiv p \wedge \sim q$, which is stated as

$x \in A \cap B$ and ($x \notin A$ or $x \notin B$).

115.(B)

The plane passes through the point (0, 7, –7) and contains the line $\dfrac{x+1}{-3} = \dfrac{y-3}{2} = \dfrac{z+2}{1}$.

$\therefore$ Required equation of the plane is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1-0 & 3-7 & -2+7 \\ -3 & 2 & 1 \end{vmatrix} = 0 \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -4 & 5 \\ -3 & 2 & 1 \end{vmatrix} = 0$$

$\therefore\ \hat{i}(-4-10) - \hat{j}(-1+15) + \hat{k}(-2-12) = 0$

$\therefore\ -14\hat{i} - 14\hat{j} - 14\hat{k} = 0 \Rightarrow \hat{i} + \hat{j} + \hat{k} = 0$

Hence Cartesian equation of the plane is $x + y + z = 0$

116.(B)

$|\bar{e}_1 + \bar{e}_2|^2 = |\bar{e}_1^2| + |\bar{e}_2^2| + 2\bar{e}_1\bar{e}_2 \cos\theta$

Here $|\bar{e}_1| = 1$, $|\bar{e}_2| = 1$ and $|\bar{e}_1 + \bar{e}_2| = 1$

$\therefore\ (1) = (1)^2 + (1)^2 + 2(1)(1)\cos\theta$

$\therefore\ \dfrac{-1}{2} = \cos\theta \Rightarrow \theta = 120°$

117.(D)

$$y = \log\tan\left(\frac{x}{2}\right) + \sin^{-1}(\cos x)$$

$$= \log\tan\left(\frac{x}{2}\right) + \sin^{-1}\left[\sin\left(\frac{\pi}{2} - x\right)\right] = \log\tan\left(\frac{x}{2}\right) + \frac{\pi}{2} - x$$

$$\therefore \frac{dy}{dx} = \frac{1}{\tan\left(\frac{x}{2}\right)} \times \sec^2\left(\frac{x}{2}\right) \times \left(\frac{1}{2}\right) + 0 - 1 = \frac{\cos\frac{x}{2}}{\sin\frac{x}{2}} \times \frac{1}{\cos^2\frac{x}{2}} \times \frac{1}{2} - 1$$

$$= \frac{1}{2\sin\frac{x}{2}\cos\frac{x}{2}} - 1 = \frac{1}{\sin x} - 1 = \operatorname{cosec} x - 1$$

118.(D)

$\bar{a}$ is ⊥er to $\bar{b} + \bar{c} \Rightarrow \bar{a}\cdot(\bar{b}+\bar{c}) = 0 \Rightarrow \bar{a}\cdot\bar{b} + \bar{a}\cdot\bar{c} = 0$ ...(1)

$\bar{b}$ is ⊥er to $(\bar{c}+\bar{a}) \Rightarrow \bar{b}\cdot(\bar{c}+\bar{a}) = 0 \Rightarrow \bar{b}\cdot\bar{c} + \bar{b}\cdot\bar{a} = 0$ ...(2)

$\bar{c}$ is ⊥er to $(\bar{a}+\bar{b}) \Rightarrow \bar{c}\cdot(\bar{a}+\bar{b}) = 0 \Rightarrow \bar{c}\cdot\bar{a} + \bar{c}\cdot\bar{b} = 0$ ...(3)

From (1), (2) and (3), we get

$2(\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a}) = 0$ ...(4)

Now $|\bar{a}+\bar{b}+\bar{c}|^2 = |\bar{a}|^2 + |\bar{b}|^2 + |c|^2 + 2(\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a})$

$\therefore |\bar{a}+\bar{b}+\bar{c}|^2 = (2)^2 + (3)^2 + (4)^2 + 0$ ...[From (4) and data given]

$= 4 + 9 + 16 = 29$

$\therefore |\bar{a}+\bar{b}+\bar{c}| = \sqrt{29}$

119.(D)

Let $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4} = \lambda$ and $\frac{x-3}{1} = \frac{y-k}{2} = \frac{z}{1} = \mu$

Since given lines intersect, we write

$2\lambda + 1 = \mu + 3$ ...(1)

$3\lambda - 1 = 2\mu + k$ ...(2)

$4\lambda + 1 = \mu$ ...(3)

Substituting value of μ from (3) in (1), we get

$2\lambda + 1 = (4\lambda + 1) + 3 \Rightarrow 2\lambda = -3 \Rightarrow \lambda = \frac{-3}{2}$

$\therefore \mu = 4\left(\frac{-3}{2}\right) + 1 = -6 + 1 = -5$

Substituting values of λ and μ in (2), we get

$3\left(\frac{-3}{2}\right) - 1 = 2(-5) + k \Rightarrow k = \frac{9}{2}$

120.(A)

Here $\begin{vmatrix} 1 & 2 & x \\ 4 & -1 & 7 \\ 2 & 4 & -6 \end{vmatrix} = 0$

$\therefore\ (6 - 28) - 2(-24 - 14) + x(16 + 2) = 0$

$\therefore\ -22 + 76 + 18x = 0 \quad \Rightarrow x = -3$

121.(D)

$\frac{dy}{dx} = \frac{x+y+1}{x+y-1}$

Put $x + y = u \Rightarrow 1 + \frac{dy}{dx} = \frac{du}{dx}$

$\therefore\ \frac{du}{dx} - 1 = \frac{u+1}{u-1} \quad \Rightarrow \frac{du}{dx} = \frac{u+1}{u-1} + 1 = \frac{2u}{u-1}$

$\therefore\ \left(\frac{u-1}{u}\right) du = 2dx$

Integrating both sides, we get

$\int du - \int \frac{du}{u} = \int 2dx$

$\therefore\ u - \log u = 2x + c$

$\therefore\ x + y - \log(x + y) = 2x + c \quad \Rightarrow y = x + \log(x + y) + c$

122.(C)

$p \wedge (\sim p \vee \sim q) \wedge q = p \wedge q \wedge \sim(p \wedge q)$

$= (p \wedge q) \wedge \sim(p \wedge q) \equiv F$

123.(D)

Equation of required line is

$\frac{x-3}{1-3} = \frac{y-4}{-1-4} = \frac{z+7}{6+7} = \lambda$ ...(Say)

$\therefore\ \frac{x-3}{-2} = \frac{y-4}{-5} = \frac{z+7}{13} = \lambda$

$\therefore\ x = -2\lambda + 3,\ y = -5\lambda + 4,\ z = 13\lambda - 7$

124.(B)

$(1 + i)^5 (1 - i)^7$

$= (1 + i)^5 (1 - i)^5 (1 - i)^2 = [(1 + i)(1 - i)]^5 (1 - 2i + i^2)$

$= (1 - i^2)^5 (-2i) = (2)^5 (-2i) = -64\,i$ ...[$\because i^2 = -1$]

125.(B)

Expected value $= \Sigma p_i x_i$

$= (0)(0.35) + (500)(0.25) + (1000)(0.15) + (1500)(0.1) + (2000)(0.08)$
$+ (2500)(0.05) + (3000)(0.02)$

$= 0 + 125 + 150 + 150 + 160 + 125 + 60 = 770$

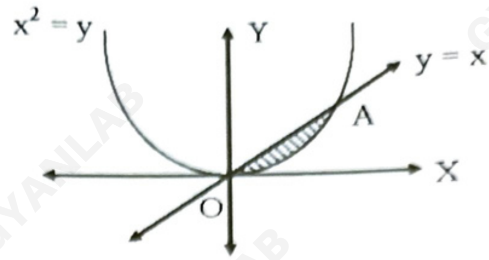
126.(B)

Points of intersection of given curves are

$x^2 - x = 0 \Rightarrow x(x-1) = 0 \quad \Rightarrow x = 0, 1$

i.e. O(0, 0) and A(1, 1)

Required area is shaded.



$$\therefore \; A = \int_0^1 \left(x - x^2\right) dx$$

$$= \left[\frac{x^2}{2}\right]_0^1 - \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3} = \frac{1}{6} \text{ sq. units.}$$

127.(A)

$a_{11} A_{11} + a_{12} A_{12} + a_{13} A_{13} = |A|$

$$\therefore \; |A| = \begin{vmatrix} 3 & 2 & 4 \\ 1 & 2 & 1 \\ 3 & 2 & 6 \end{vmatrix}$$

$$= 3(12-2) - 2(6-3) + 4(2-6) \quad = 30 - 6 - 16 = 8$$

128.(A)

We have, $x + y\dfrac{dy}{dx} = \sec(x^2 + y^2)$

Put $x^2 + y^2 = u \quad \Rightarrow \; 2x + 2y\dfrac{dy}{dx} = \dfrac{du}{dx}$

$\therefore \; x + y\dfrac{dy}{dx} = \dfrac{1}{2}\dfrac{du}{dx}$

$\therefore \; \dfrac{1}{2}\dfrac{du}{dx} = \sec u$

$\therefore \; \displaystyle\int \frac{du}{\sec u} = \int 2\,dx$

$\therefore \; \sin u = 2x + c \;\; \Rightarrow \; \sin(x^2 + y^2) = 2x + c$

129.(C)

$$\lim_{x\to\infty} \sqrt{x^2 + 5x - 7} - x$$

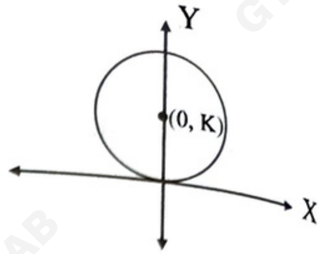
$$= \lim_{x\to\infty} \frac{\left(\sqrt{x^2+5x-7} - x\right)\left(\sqrt{x^2+5x-7} + x\right)}{\left(\sqrt{x^2+5x-7} + x\right)} \quad = \lim_{x\to\infty} \frac{x^2 + 5x - 7 - x^2}{\left(\sqrt{x^2+5x-7} + x\right)}$$

Dividing numerator and denominator by x, we get

$$= \lim_{x\to\infty} \frac{5 - \frac{7}{x}}{\left(\sqrt{1 + \frac{5}{x} - \frac{7}{x^2}} + 1\right)} = \frac{5}{\sqrt{1} + 1} = \frac{5}{2}$$

130.(A)

Let (0, K) be the centre of the circle.



Since the circle passes through origin, we write

$(x-0)^2+(y-K)^2=K^2$

$\therefore\ x^2+y^2-2Ky+K^2=K^2$

$\therefore\ x^2+y^2-2Ky=0$ ...(1)

Differentiating w.r.t. x, we get

$$2x+2y\frac{dy}{dx}-2K\frac{dy}{dx}=0$$

$\therefore\ K=\dfrac{x+y\frac{dy}{dx}}{\frac{dy}{dx}}$ and substituting value of K in (1), we get

$$x^2+y^2-2\left(\frac{x+y\frac{dy}{dx}}{\frac{dy}{dx}}\right)y=0$$

$$\therefore\ \left(x^2+y^2\right)\frac{dy}{dx}-2xy-2y^2\frac{dy}{dx}=0 \Rightarrow \left(x^2-y^2\right)\frac{dy}{dx}-2xy=0$$

131.(D)

Let the angles be x, 2x, 3x

$\therefore\ x+2x+3x=180° \Rightarrow 6x=180° \Rightarrow x=30°$

Thus angles of the triangle are 30°, 60°, 90°.

Now, $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}$

$$\therefore\ \frac{a}{\sin 30°}=\frac{b}{\sin 60°}=\frac{c}{\sin 90°} \Rightarrow \frac{a}{\left(\frac{1}{2}\right)}=\frac{b}{\left(\frac{\sqrt{3}}{2}\right)}=\frac{c}{(1)}$$

$$\therefore\ 2a=\frac{2b}{\sqrt{3}}=c \Rightarrow a=\frac{c}{2} \text{ and } b=\frac{\sqrt{3}\,c}{2}$$

$$\therefore\ a:b:c=\frac{c}{2}:\frac{\sqrt{3}\,c}{2}:c=1:\sqrt{3}:2$$

132.(C)

From given data, we write

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{vmatrix}=0$$

$\therefore\ \hat{i}(6\mu-27\lambda)-\hat{j}(2\mu-27)+\hat{k}(2\lambda-6)=0$

$\therefore\ 6\mu-27\lambda=0$ ...(1)

$27-2\mu=0$ ...(2)

$2\lambda-6=0$ ...(3)

From (2) and (3), we get $\mu=\dfrac{27}{2}$ and $\lambda=3$.

These values of λ & μ satisfy eq. (1)

133.(C)

We have $f(x) = [x]$

Let $[x] = K$, an integer.

$\therefore \lim_{x \to K^+} f(x) = K$ and $\lim_{x \to K^-} f(x) = K - 1$

Thus given function is not continuous at all integral values in its domain.

$\because$ f is discontinuous at $x = 0, 1$.

134.(C)

Direction ratios of a line joining (3, 1, 4) and (7, 2, 12) are 4, 1, 8

Let $(a_1, b_1, c_1) = (2, 2, 1)$ and $(a_2, b_2, c_2) = (4, 1, 8)$.

Hence angle θ between the lines is given by

$$\cos\theta = \frac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1}\sqrt{a_2^2 + b_2^2 + c_2^2}}$$

$$= \frac{8+2+8}{\sqrt{4+4+1}\cdot\sqrt{16+1+64}} = \frac{18}{\sqrt{9}\cdot\sqrt{81}} = \frac{18}{(3)(9)} = \frac{2}{3}$$

$$\therefore \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

135.(D)

$y = 4xe^x$

$$\therefore \frac{dy}{dx} = 4xe^x + 4e^x$$

$$\therefore \left(\frac{dy}{dx}\right)_{\left(-1,\frac{-4}{e}\right)} = 4(-1)\,e^{-1} + 4e^{-1} = \frac{-4}{e} + \frac{4}{e} = 0$$

Thus tangent is parallel to X axis.

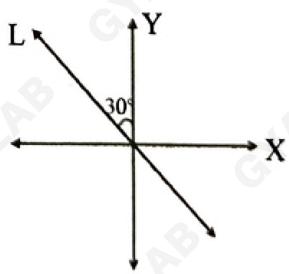
Hence required equation of tangent is $y = \frac{-4}{e}$

136.(B)

Refer figure.

Angle made by line L with positive direction of X axis is $(90° + 30°)$ i.e. 120°.

$\therefore$ Slope of line L = $\tan(120°) = \tan(\pi - 60°) = -\tan 60° = -\sqrt{3}$



137.(C)

$$f(x) = \frac{1}{\sqrt{x + |x|}}$$

Here $x + |x| \geq 0$ and $\sqrt{x + |x|} \neq 0$

$\therefore x + |x| > 0$

Now when $x > 0$, $x + |x| = x + x \Rightarrow 2x > 0$.

When $x < 0$, $x + |x| = x - x = 0$

$\therefore x > 0$ is the required domain.

138.(D)

The word ABRACADABRA has

A : 5 times, B : 2 times, R : 2 times, C, D : 1 time each

When all vowels are together, we have to arrange 7 elements.

$\therefore$ No. of arrangements $= \dfrac{7!}{2!2!} = 1260$

139.(D)

We have, $\dfrac{1}{\sqrt{2}}\tan^{-1}\left[\dfrac{f(x)}{\sqrt{2}}\right]+c \quad = \int \dfrac{1+x^2}{1+x^4}dx$ ...(1)

Let $\quad I = \int \dfrac{1+x^2}{1+x^4}dx$

Dividing numerator and denominator by x^2, we get

$$I = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x^2+\frac{1}{x^2}\right)}dx = \int \frac{\left(1+\frac{1}{x^2}\right)}{\left(x-\frac{1}{x}\right)^2+2}dx$$

Put $x - \dfrac{1}{x} = t \Rightarrow \left(1+\dfrac{1}{x^2}\right)dx = dt$

$$\therefore\ I = \int \frac{dt}{t^2+2} = \int \frac{dt}{(t)^2+\left(\sqrt{2}\right)^2}$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\left[\frac{(t)}{\sqrt{2}}\right]+c \quad = \frac{1}{\sqrt{2}}\tan^{-1}\left[\frac{\left(x-\frac{1}{x}\right)}{\sqrt{2}}\right]+c \qquad ...(2)$$

From (1) and (2), $f(x) = x - \dfrac{1}{x}$

140.(B)

$\sin 90° = \sin 5(18°)$ and let $18° = A$

$\sin 90° = \sin 5A = \sin(3A + 2A)$

$\therefore\ 90° = 3A + 2A \quad \Rightarrow \sin(90° - 3A) = \sin 2A$

$\therefore\ \sin 2A = \cos 3A \quad \Rightarrow 2\sin A\cos A = 4\cos^3 A - 3\cos A$

$\therefore\ \cos A\,(2\sin A - 4\cos^2 A + 3) = 0$

$\therefore\ \cos A = 0$ or $[2\sin A - 4(1-\sin^2 A) + 3] = 0$

$\therefore\ A = \dfrac{\pi}{2}$ or $\quad 4\sin^2 A + 2\sin A - 1 = 0$

Since, $A = 18°$, $A \neq \dfrac{\pi}{2}$

$\therefore\ 4\sin^2 A + 2\sin A - 1 = 0$

$\therefore\ \sin A = \dfrac{-2\pm\sqrt{4+16}}{2(4)} = \dfrac{-2\pm\sqrt{20}}{8} = \dfrac{-2\pm2\sqrt{5}}{8} = \dfrac{-1\pm\sqrt{5}}{4}$

$\therefore \sin A = \frac{-1+\sqrt{5}}{4}$ or $\sin A = \frac{-1-\sqrt{5}}{4}$

Since, $\sin A > 0$, $\sin A = \frac{\sqrt{5}-1}{4}$

141.(D)

$4\sin^{-1}x + 6\cos^{-1}x = 3\pi$

$\therefore 4(\sin^{-1}x + \cos^{-1}x) + 2\cos^{-1}x = 3\pi$

$\therefore 4\left(\frac{\pi}{2}\right) + 2\cos^{-1}x = 3\pi \Rightarrow 2\cos^{-1}x = \pi$

$\therefore \cos^{-1}x = \frac{\pi}{2} \Rightarrow x = \cos\frac{\pi}{2} = 0$

142.(B)

$h(x) = \sqrt{4f(x)+3g(x)}$ ...(1)

$\therefore h(1) = \sqrt{4(4)+3(3)} = 5$ ...(2) ...[From data given]

Squaring (1), we get

$[h(x)]^2 = 4f(x) + 3g(x)$

Differentiating w.r.t. x , we get

$2\,h(x)\,h'(x) = 4f'(x) + 3g'(x)$

At $x = 1$, we get

$2(5)\,h'(1) = 4(3) + 3(4) = 24$ ...[From (2) and data given]

$\therefore h'(1) = \frac{24}{10} = \frac{12}{5}$

143.(C)

Line $\frac{x+1}{2} = \frac{y-m}{3} = \frac{z-4}{6}$ lies in plane $3x - 14y + 6z + 49 = 0$

$\therefore$ Point $(-1, m, 4)$ lies in the plane.

$\therefore 3(-1) - 14(m) + 6(4) + 49 = 0 \Rightarrow -3 - 14m + 24 + 49 = 0$

$\therefore 14m = 70 \Rightarrow m = 5$

144.(A)

$\sin x \cos x = \frac{1}{4}$

$\therefore 2\sin x \cos x = \frac{2}{4} = \frac{1}{2} \Rightarrow \sin 2x = \frac{1}{2} = \sin 30°$

$\therefore 2x = 30° = \frac{\pi}{6} \Rightarrow x = \frac{\pi}{12}$

$\therefore x = \frac{\pi}{2} \pm \frac{\pi}{12} \Rightarrow x = \frac{\pi}{2} + \frac{\pi}{12}$ or $x = \frac{\pi}{2} - \frac{\pi}{12}$

$\therefore x = \frac{7\pi}{12}$ or $\frac{5\pi}{12} \Rightarrow x = \frac{\pi}{12}, \frac{5\pi}{12}$ .... $\left[\because x \in \left(0, \frac{\pi}{2}\right)\right]$

145.(C)

$x = a\cos\theta,\ y = b\sin\theta$

$\therefore \dfrac{dx}{d\theta} = -a\sin\theta,\ \dfrac{dy}{d\theta} = b\cos\theta$

$\therefore \dfrac{dy}{dx} = \dfrac{b\cos\theta}{-a\sin\theta} = \left(\dfrac{-b}{a}\right)\cot\theta$

$$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{d\theta}\left(\frac{dy}{dx}\right)\cdot\frac{d\theta}{dx}$$

$$= \frac{d}{d\theta}\left[\left(\frac{-b}{a}\right)\cot\theta\right]\times\frac{1}{\left(\frac{dx}{d\theta}\right)}$$

$$= \frac{\left(\frac{-b}{a}\right)\left(-\operatorname{cosec}^2\theta\right)}{-a\sin\theta} = \left(\frac{-b}{a}\right)\times\frac{1}{a}\times\frac{1}{\sin^3\theta}$$

$$\therefore \left(\frac{d^2y}{dx^2}\right)_{\theta=\frac{\pi}{4}} = \left(\frac{-b}{a^2}\right)\left[\frac{1}{\left(\sin\frac{\pi}{4}\right)^3}\right] = \left(\frac{-b}{a^2}\right)\left(\sqrt{2}\right)^3 = -2\sqrt{2}\left(\frac{b}{a^2}\right)$$

146.(A)

$$\text{Let } I = \int\frac{x+\sin x}{1+\cos x}dx = \int\frac{x+\sin x}{2\cos^2\frac{x}{2}}dx$$

$$= \int\frac{x}{2\cos^2\frac{x}{2}}dx + \int\frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2\cos^2\frac{x}{2}}dx = \frac{1}{2}\int x\sec^2\frac{x}{2}dx + \int\tan\frac{x}{2}dx$$

$$= \frac{1}{2}\left[x\tan\frac{x}{2}(2) - \int 2\tan\frac{x}{2}dx\right] - 2\log\left|\cos\frac{x}{2}\right| + c$$

$$= x\tan\frac{x}{2} + 2\log\left|\cos\frac{x}{2}\right| - 2\log\left|\cos\frac{x}{2}\right| + c = x\tan\frac{x}{2} + c$$

147.(C)

Half the quantity of ice melts in 20 minutes and x_0 is the initial quantity of ice.

$\therefore$ Quantity after 20 minutes $= \dfrac{x_0}{2}$

Quantity after 40 minutes $= \dfrac{1}{2}\left(\dfrac{x_0}{2}\right) = \dfrac{x_0}{4}$

148.(B)

$$\text{Let } I = \int_0^{\frac{\pi}{2}}\frac{dx}{5+4\sin x}$$

Put $\tan\dfrac{x}{2} = t \Rightarrow \sec^2\dfrac{x}{2}\left(\dfrac{1}{2}\right)dx = dt$

$\therefore\ dx = \frac{2dt}{1+t^2}$ and $\sin x = \frac{2t}{1+t^2}$

When $x = 0, t = 0$ and when $x = \frac{\pi}{2}, t = 1$

$$\therefore\ I = \int_0^1 \frac{1}{5+4\left(\frac{2t}{1+t^2}\right)} \times \frac{2dt}{1+t^2} = 2\int_0^1 \frac{dt}{5+5t^2+8t} = \frac{2}{5}\int_0^1 \frac{dt}{t^2+\frac{8}{5}t+1}$$

$$= \frac{2}{5}\int_0^1 \frac{dt}{t^2+\frac{8}{5}t+\frac{16}{25}+\frac{9}{25}} = \frac{2}{5}\int_0^1 \frac{dt}{\left(t+\frac{4}{5}\right)^2+\left(\frac{3}{5}\right)^2}$$

$$\therefore\ I = \frac{2}{5} \times \frac{1}{\left(\frac{3}{5}\right)}\left[\tan^{-1}\left[\frac{t+\frac{4}{5}}{\left(\frac{3}{5}\right)}\right]\right]_0^1 = \frac{2}{3}\left[\tan^{-1}\left(\frac{5t+4}{3}\right)\right]_0^1$$

$$= \frac{2}{3}\left[\tan^{-1}3 - \tan^{-1}\frac{4}{3}\right] = \frac{2}{3}\tan^{-1}\left[\frac{3-\left(\frac{4}{3}\right)}{1+3\left(\frac{4}{3}\right)}\right]$$

$$= \frac{2}{3}\tan^{-1}\left(\frac{5}{3}\times\frac{1}{5}\right) = \frac{2}{3}\tan^{-1}\left(\frac{1}{3}\right)$$

Comparing with given data, we get

$A = \frac{2}{3}, B = \frac{1}{3} \Rightarrow A + B = 1$

149.(A)

We have $n = 4$ and $P(x = 0) = \frac{16}{81}$

$\therefore\ \frac{16}{81} = {}^4C_0(p)^4(q)^0$

$\therefore\ \frac{16}{81} = (p)^4 \quad \Rightarrow P = \frac{2}{3} \quad \Rightarrow q = \frac{1}{3}$

$\therefore\ P(x = 4) = {}^4C_4(p)^0(q)^4 = (1)(1)\left(\frac{1}{3}\right)^4 = \frac{1}{81}$

150.(B)

We have, $3x - 4y + 4 = 0$ …(1) and $6x - 8y - 7 = 0 \Rightarrow 3x - 4y - \frac{7}{2} = 0$ …(2)

Lines (1) and (2) are parallel to one another and these lines are tangents to the circle.

$\therefore$ Distance between the lines is equal to diameter of circle.

$$\therefore\ D = \frac{\left|4-\left(\frac{-7}{2}\right)\right|}{\sqrt{(3)^2+(-4)^2}} = \frac{\left(4+\frac{7}{2}\right)}{\sqrt{25}} = \frac{15}{2(5)} = \frac{3}{2} \Rightarrow \text{radius} = \frac{3}{4}\text{ units}$$